The question of stability is an important one in the study of celestial mechanics. An example of this is our solar system. The planets evolve around the Sun, but must this always be so? Can a small perturbation of our system, for instance, due to a gravitational influence of some object passing nearby, lead to the destruction of our equilibrium?

In 1964, Vladimir Igorevich Arnold, who is one of the fathers of the field of modern dynamical systems, has published a paper containing an intriguing example. This discussed a stable system. (In the language of classical mechanics, it was fully integrable.) In such setting, one expects the system to be stable under a small perturbation. Nevertheless, the presented example behaved as if it was unstable. Arnold has in fact proved, that under arbitrarily small perturbation we can have changes of energy by a fixed amount, that is independent from the size of the perturbation. Moreover, these changes behaved chaotically.

What does this have to do with celestial mechanics? Arnold has conjectured that the diffusion mechanism from his example (today known as Arnold diffusion), should be present in the $n$-body problem. In particular, this means that we should find such phenomena in our solar system. The conjecture has been posed half a century ago, but to this day has not been proven.

Even though we do not have a formal proof, there are a number of results that confirm existence of Arnold diffusion in our solar system. Some of these are associated with the asteroid belt, which is located between the orbits of Mars and Jupiter, or with Lyapunov orbits between Jupiter and Sun. The mechanism leading to diffusion is well known and understood, and is described in a number of papers (some of them have been co-authored by participants of this proposal). The main problem in turning these into a complete and rigorous proof is the lack of analytic formulae for the trajectories of the $n$-body problem.

In the research project that is the subject of this grant proposal, we aim to use computer assisted estimates instead of analytic solutions. For the past twenty years we can observe a dynamic development in the field of computer assisted proofs. This includes their applications to the $n$-body problem. By using topological, geometric or functional-analytic theorems, which are accompanied by rigorous, interval arithmetic based verifications of their assumptions, one can prove numerous properties of dynamical systems. We plan to use these tools to verify assumptions of our mechanism that leads to Arnold diffusion in the $n$-body problem.