(Anti)Classification and invariants in dynamics

Abstract for the general public

In mathematics, dynamical systems is the research field which investigates systems that evolve in time according to a given rule. In this regard, a dynamical system is a fundamental structure in mathematics which also allows for a wide variety of physical systems to be modeled by. For this reason, dynamical systems spread far into both the theoretical and applied side of mathematics.

A fundamental theme in dynamics is to classify systems up to appropriate equivalence relations that preserve significant qualitative dynamical behavior. For this purpose, one seeks for *dynamical invariants* that are easy to compute and help to determine whether two systems can be equivalent to each other. They usually reflect different kinds of dynamical behavior and complexity of a system. In the best case scenario, these invariants can be expressed as a single number. While one often can find invariants that are preserved under equivalence, even within specific classes of dynamical systems it turns out to be very hard to find *complete invariants*, that is, invariants that agree for two systems if and only if the systems are equivalent to each other. In this research project we aim at finding and studying new invariants for particular classes of dynamical systems, often associated with behavior of so-called *low complexity*. The systems under consideration combine some topological and measure-theoretical properties. Hence, our project lies at the interface of topological dynamics and ergodic theory.

Historically motivated by problems in statistical mechanics, ergodic theory examines statistical properties of dynamical systems. In particular, one is interested in the long-term behavior of the system as well as the relationship between its time and space averages. Nowadays, ergodic theory is a powerful amalgam of mathematical methods used for the analysis of statistical properties of dynamical systems and is a large and rapidly developing subject with fruitful connections with several other fields of mathematics (especially differential geometry, probability theory and number theory). For these reasons, the problems and methods of ergodic theory interest not only mathematicians, but also the researchers in physics, biology, meteorology etc.

Given the difficulty of classification, starting from the late 1990's so-called *anti-classification results* in ergodic theory have emerged. Using concepts from descriptive set theory, they show in a rigorous way that classification for broad classes of measure-preserving dynamical systems is impossible. In this research project, we plan to address several new questions that arise from those results. On the one hand, we plan to establish further anti-classification results within more restricted classes of measure-preserving systems. On the other hand, we aim at extending anti-classification results to the topological category.