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Towards inconsistent Dunn semantics

Abstract for the general public

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Given a sentence 'S' and two truth values, true and false, S is typically evaluated in two ways: it is true or it is false. That way of evaluating sentences is pretty standard, and it is at the core of most semantics for formal languages. When there are only two semantic evaluations —viz. being (just) true or being (just) false— and the vocabulary is interpreted in a certain way, certain argument forms get validated and other get invalidated, giving rise to what is known as "classical logic".

However, in full generality, a sentence can be also *both true and false* or *neither true nor false*. Allowing for more relations between sentences and truth values is the key insight behind *Dunn semantics*. Allowing for more semantic valuations beyond being (just) true or being (just) false, even maintaining the interpretation of the vocabulary, is a way to validate and invalidate different argument forms than in the classical case. Some non-classical logics are obtained or at least commonly presented in that way.

Dunn semantics is a simple yet very flexible framework that allows modeling a significant number of already known logics, and also the systematic crafting of new logics. However, and although Dunn semantics allows for a certain amount of inconsistency in the languages it models since some sentences can be both true and false, Dunn semantics itself is entirely consistent: no proposition is both true and not true, or both false and not false; no argument is both logically valid and logically invalid. Also, Dunn semantics allows for some incompleteness in the languages it models, since some sentences can be neither true nor false, Dunn semantics is entirely complete: all propositions are either true or not true, and either false or not false; and all arguments are either logically valid or logically invalid. This is a problem for some really committed non-classicalists, and for some classicalists who expect non-classicalists to be really committed. For them, inconsistency and incompleteness should go all the way up from the languages modeled to the semantics itself.

Another way of seeing the pressure for a non-classical version of Dunn semantics is that there are logical notions involved in the semantics. Consider the expressions "The sentence S is both true and false" or "The sentence S is neither true nor false" —or the latter in a more explicit form: "The sentence S is not true and is not false"—. Even more importantly, the definition of logical invalidity is as follows: An argument is logically invalid if and only if there is an interpretation in which the premises are true but the conclusion is not true. All these expressions include logical vocabulary, like the particles 'and', 'not' or 'if', and what are their specific properties will vary from a supporter of classical logic to a supporter of a non-classical logic. If the non-classical logician treats them classically, they must at least explain why such a double standard is adopted; if not, a non-classical meta-theory must be developed.

Although there are some notable advances in the topic of non-classical meta-theories, the field is virtually unexplored. Moreover, the case of Dunn semantics is particularly challenging since a non-classical treatment of negation in semantic notions, a treatment according to what motivated Dunn semantics in the first place might re-shape Dunn semantics in unexpected and even unwelcomed ways. This project aims at filling that lacuna by providing a non-classical understanding of some key semantic notions, such as satisfiability, and those that rest upon them, like the notion of invalidity. For definiteness, the following questions will be addressed:

Q1. Can a proposition be both true and not true?

Q2. Can an argument be both logically valid and logically invalid?

Q3. Suppose that **L** is among one's preferred logics, but **L*** is not. Do the connectives in **L*** can make sense from the perspective of **L**?

Q4. How to understand negative expressions like "S is not true" or "S is not false"? Are they best analyzed in terms of positive expressions atoms and some negation, or positive and negative expressions should be taken as equally basic?

The overarching idea to answer these questions is this: just as truth and falsity were given a conceptually independent treatment in Dunn semantics, i.e. falsity is not definable in terms of truth nor vice versa, the elements of other pairs of "polar" notions, such as satisfiable/unsatisfiable or valid/invalid, must be given a similar independent treatment.