

Nonreflexive Function Spaces: Fourier v. Martingale approach

Harmonic analysis emerged as a mathematical tool for investigating physical phenomena of oscillatory nature. During the last few centuries it proved extremely useful at that, while being developed into an essentially self-contained branch of mathematics. It is impossible to list all - often unexpected - applications of harmonic analysis in other disciplines of mathematics. These include beautiful results in number theory, additive combinatorics, probability theory, differential equations, dynamical systems, potential theory, geometry of infinite dimensional spaces, or even topology. In the twentieth century we witnessed rapid progress in harmonic analysis, partly thanks to the theory of singular integral operators called Calderon-Zygmund operators. Today, our knowledge about them is enormous and they have a wide range of applications.

Martingale theory is a central and important branch of probability theory. Initially it was developed as a tool for analyzing winning strategies in game theory. Over the years it turned out that martingale theory is extremely useful in many branches of mathematical analysis - in particular in harmonic analysis and in the construction of solutions of partial differential equations. One of the prime examples of martingales, the Wiener process also called Brownian motion, found a wide range of applications, not only in mathematics but also in theoretical physics, social sciences and economy. Within mathematics, the classical martingale theory is well suited to study spaces of integrable functions. However it is not as effective when it comes to study singularities of measures, which are solutions of partial differential equations and their natural generalizations.

In this project, we use modern martingale theory to study Banach spaces of differentiable functions, which play a central role in analysis. They are crucial in the theory of differential equations, dynamical systems, probability theory etc. However their inner structure, especially in the non-reflexive case, remains largely elusive. This is true in particular when comparing Banach spaces of differentiable functions to Banach spaces of analytic functions.

One of the main goals of this project is to expand the knowledge of the structure of non-reflexive Banach spaces of differentiable functions and description of singularities of their elements. In the project we would like to adapt and develop martingale methods to study singularities of differentiable functions and their generalizations. We want to connect these with embeddings and trace theorems. Moreover we want to exploit the possible similarities and differences between the theories of analytic and smooth functions.

Harmonic analysis of operators in non-reflexive spaces is not only important and crucial in applications, but it is also a task full of mathematical beauty and leads to deep often unexpected links, and therefore it should most certainly be studied and developed intensively.