

One of ever-present ideas in mathematics is the idea of infinity. At each step one encounters things which are infinitely many — starting from numbers, sets, functions and so on. Having an infinite amount of possible numbers which are needed to describe a real world problem is a typical situation. One says that a variable can assume infinitely many values. This setting is sufficient to describe a position of a particle on a line or a plane pendulum. In order to have more freedom of motion though one needs to take several variables of that type. It allows us for example to describe a motion of a planet in the Solar System (with 6 variables) or a rigid body (with 12 variables). These problems possess usually extra structures, for example geometrical or differential which are crucial in the process of finding solutions and describing their behaviour. The typical mathematical concept needed in this framework is a smooth  $n$ -dimensional manifold. It can be seen as a generalization of a notion of a surface in space to arbitrary dimension. Additionally one uses the notion of a Lie group to describe symmetries of the problem.

This approach however is not sufficient to formulate more complicated problems present in contemporary science. For example problems in quantum mechanics or hydrodynamics require an infinite number of variables and new structures are needed. We say that spaces where these systems live are infinite dimensional. Instead of geometry, one usually employs functional analysis which is a branch of mathematics dealing with such spaces. The idea to consider infinite dimensional geometry is not new but in recent years there is a trend renewing the interest in this area. One of the topics of research is related to the so-called Poisson structures on infinite dimensional manifolds which are a tool which allows an elegant construction of equations and integrals of motion for a system. However taking a tool out of the world of finite dimensional geometry and attempting to apply it in the world of infinite dimensional geometry is often very tricky. Straightforward approach usually fails and unexpected problems present themselves. It is often a challenge to find good non-trivial examples and counter-examples.

There are not so many integrable (it means that we can find a solution) systems given in a mathematically rigorous settings. Most of the integrable systems are given only in a formal way and lack precise geometrical picture. One of the first systems to benefit from geometrical approach was a Korteweg–de Vries equation describing solitary waves traveling without dissipation in the shallow water (so-called solitons). The geometrical object used by Segal and Wilson in 1985 for description of this system was restricted Grassmannian.

The aim of this project is analysis of Poisson structures and new integrable associated to restricted Grassmannian using modern geometrical tools. Previous results by in this area already uncovered some interesting structures and a hierarchy of integrable equations. We hope that further study would allow us to strengthen the understanding of the geometry and to find new links to other known problems.