The general aim of the project is to study combinatorial structures in context of set theory, topology and functional analysis. Roughly speaking, in mathematics one can deal with two kind of objects: definable ones (such as Borel sets or, in general, sets which can be defined with an explicit formula) and more complicated structures which do not have any definable description. The structure of our project reflects this division. The first part concerns definable structures: analytic P-ideals and Banach spaces with unconditional bases, while the second concerns ultrafilters, non-separable Banach spaces and other objects far from being definable.

Both parts have concrete motivations standing behind. In the case of the first part this is the close connection between analytic P-ideals and Banach spaces with unconditional bases, discovered by Borodulin-Nadzieja and Farkas. This connection gives a hope that the theory of Banach spaces will provide an interesting structure results and examples in the theory of analytic P-ideals and vice versa. The motivation for the second part is a strong conviction that measure algebras has not been studied enough as a source of important combinatorial objects.

We want to investigate interactions between combinatorics of families of finite sets on the natural numbers, geometry of the associated Banach spaces, and complexity of the analytic P-ideals generated by the bases of these spaces. In particular we would like to characterize families of finite sets inducing the Banach spaces which are ℓ_1-saturated or which have the Schur property, investigate ideals which are induced by non-compact families of sets and study the related universality questions.

We plan to investigate the structure of the measure algebras from the point of view of combinatorics, topology and forcing. In particular we would like to investigate certain subalgebras of the measure algebra defined in terms of approximation with clopens, use methods introduced by Borodulin-Nadzieja and Sobota in to study ultrafilters in the random model, and to examine the relationship between $\mathcal{P}(\omega)/Fin$ and the measure algebra.

We use the standard methods of pure mathematics, i.e. attempting to prove or refute conjectures, taking into account relevant results from the literature and discussing problems with colleagues to benefit from new ideas in the field. These discussions may result in the extension of our research into unexpected directions.

In both parts we are going to use methods discovered very recently to attack some known open problems and to study structures of certain classical objects.

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