Wider research context

Set theory is the mathematical study of infinity and has been very successful both in answering its own deep fundamental questions and in being applied elsewhere. Gdel's Program is a major set theoretic program addressing the most fundamental set theoretic issue: independence. One important aspect of Gdel's Program is the identification of the core of the universe, i.e., definable objects whose definability persists through all generic extensions of the universe. In this project we study objects in the core that are grounded in the universally Baire sets.

Objectives

This proposal concentrates on objects that are benign in their effect on the Derived Model, which is a model of determinacy derived from a cardinal that is a limit of Woodin cardinals and which contains all sets of reals that are strongly determined. The global aim of this proposal is the classification of benign objects in the presence of large cardinals. One particularly intriguing possibility is that the class of all benign objects is itself a benign object. Arguably, this will be the ultimate Derived Model. Recent advances Mller and Sargsyan made independently on two different topics suggest that a systematic study of models of determinacy with a rich structure above their Theta is not only possible but in fact necessary for the advancement of inner model theory. This study is the key goal of this proposal.

Methods

We use methods from descriptive inner model theory, an area of inner model theory in which descriptive set theoretic methods are employed to advance the construction of complicated inner models. More specifically, the methods we use build on recent breakthrough results of Mller, Sargsyan with Larson and Wilson, Sargsyan with Larson, and Sargsyan with Trang.

Innovation

We aim to study models of determinacy that are not constructible above their powerset of the reals. Except for a small number of very recent results, this is a completely unexplored area. Last year, Larson and Sargsyan used such a model to refute the long-standing iterability conjecture for certified extender constructions. This indicates that understanding the internal structure of such models, in their case the Chang model, is important for the advancement of the inner model program. We propose a novel systematic analysis of such models and expect that forcing extensions of these models will have an interesting combinatorial structure. Therefore, the aims in this proposal have the potential to be applied to refute other long-standing conjectures in the inner model program. This will lead to a clearer picture of inner models in the short extender region.